MÉTODOS ESTADÍSTICOS DE LA INGENIERÍA

INTERVALOS DE CONFIANZA

- Intervalo de confianza para la media μ de una distribución normal con varianza σ^2 conocida:

$$\boxed{I_e = \left(\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)} \quad \diamondsuit\ \widehat{\mu} = \overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \quad \diamondsuit\ X_1, ..., X_n \ \ m.a.s. \text{ de } X \sim N(\mu, \sigma)$$

- Intervalo de confianza para la media μ de una distribución normal con varianza desconocida:

$$\boxed{I_e = \left(\overline{X} \pm t_{n-1;\,\alpha/2}\frac{S}{\sqrt{n}}\right)} \quad \diamondsuit\ \ \widehat{\mu} = \overline{X} \quad \diamondsuit\ \ \widehat{\sigma^2} = S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2 = \frac{\sum_{i=1}^{n} X_i^2 - n\left(\overline{X}\right)^2}{n-1}$$

$$\diamondsuit\ \ \overline{X} \sim N\left(\mu, \sigma/\sqrt{n}\right) \quad \text{y} \quad (n-1)\frac{S^2}{\sigma^2} \sim \chi^2_{n-1} \text{ son independientes} \Longrightarrow \frac{\overline{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}$$

- Intervalo de confianza para la varianza σ^2 de una distribución normal con media μ conocida:

$$\boxed{I_e = \left(\frac{nD^2}{\chi^2_{n;\,\alpha/2}}, \frac{nD^2}{\chi^2_{n;\,1-\alpha/2}}\right)} \quad \diamondsuit\ \widehat{\sigma^2} = D^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \mu)^2 \quad \diamondsuit\ n\frac{D^2}{\sigma^2} = \sum_{i=1}^{n}\left(\frac{X_i - \mu}{\sigma}\right)^2 \sim \chi^2_n$$

- Intervalo de confianza para la varianza σ^2 de una distribución normal con media desconocida:

$$\boxed{I_e = \left(\frac{(n-1)\,S^2}{\chi^2_{n-1;\,\alpha/2}}, \frac{(n-1)\,S^2}{\chi^2_{n-1;\,1-\alpha/2}}\right)} \quad \diamondsuit\ \ \widehat{\sigma^2} = S^2 \quad \diamondsuit\ \ (n-1)\frac{S^2}{\sigma^2} = \sum_{i=1}^{n}\left(\frac{X_i - \overline{X}}{\sigma}\right)^2 \sim \chi^2_{n-1}$$

- Intervalo de confianza para la diferencia de medias $\gamma = \mu_1 - \mu_2$ de dos distribuciones normales independientes con varianzas σ_1^2 y σ_2^2 conocidas:

$$\boxed{I_e = \left(\left(\overline{X} - \overline{Y}\right) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)} \quad \diamondsuit\ \ \widehat{\gamma} = \overline{X} - \overline{Y} \sim N\left(\mu_1 - \mu_2, \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$$

$$\diamondsuit\ \ \begin{cases} X_1, ..., X_{n_1} \ \ m.a.s. \text{ de } X \sim N(\mu_1, \sigma_1) \\ Y_1, ..., Y_{n_2} \ \ m.a.s. \text{ de } Y \sim N(\mu_2, \sigma_2) \end{cases} ; \qquad X \text{ e } Y \ \ \textit{independientes}$$

- Intervalo de confianza para la diferencia de medias $\mu_1 - \mu_2$ de dos distribuciones normales independientes con varianzas desconocidas e iguales ($\sigma_1^2 = \sigma_2^2 = \sigma^2$):

$$\boxed{I_e = \left(\left(\overline{X} - \overline{Y}\right) \pm t_{n_1+n_2-2;\,\alpha/2}\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}\right)} \quad \diamondsuit\ \ S_p^2 = \frac{(n_1-1)\,S_1^2 + (n_2-1)\,S_2^2}{n_1 + n_2 - 2}$$

$$\left.\begin{array}{l} \overline{X} - \overline{Y} \sim N\left(\mu_1 - \mu_2, \sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}}\right) \\ (n_1 + n_2 - 2)\,\frac{S_p^2}{\sigma^2} \sim \chi^2_{n_1+n_2-2} \end{array} \textit{indep.}\right\} \Longrightarrow \frac{\overline{X} - \overline{Y} - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$$

- Intervalo de confianza para la diferencia de medias $\gamma = \mu_1 - \mu_2$ de dos distribuciones normales independientes con varianzas desconocidas y distintas ($\sigma_1^2 \neq \sigma_2^2$):

$$\boxed{I_a = \left(\left(\overline{X} - \overline{Y}\right) \pm t_{f;\,\alpha/2}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}\right)} \quad \diamond\ f = \frac{\left(S_1^2/n_1 + S_2^2/n_2\right)^2}{\dfrac{\left(S_1^2/n_1\right)^2}{n_1+1} + \dfrac{\left(S_2^2/n_2\right)^2}{n_2+1}} - 2 \quad \diamond\ \frac{\overline{X} - \overline{Y} - \gamma}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}} \overset{a}{\approx} t_f$$

- Intervalo de confianza para el cociente de varianzas $\beta = \sigma_1^2/\sigma_2^2$ de dos distribuciones normales independientes con medias μ_1 y μ_2 conocidas:

$$\boxed{I_e = \left(\frac{D_1^2/D_2^2}{F_{n_1,n_2;\,\alpha/2}}, \frac{D_1^2/D_2^2}{F_{n_1,n_2;\,1-\alpha/2}}\right)} \quad \diamond\ \widehat{\beta} = \frac{D_1^2}{D_2^2} \quad \diamond\ \frac{D_1^2/\sigma_1^2}{D_2^2/\sigma_2^2} \sim \frac{\dfrac{\chi^2_{n_1}}{n_1}}{\dfrac{\chi^2_{n_2}}{n_2}} \equiv F_{n_1,n_2}$$

- Intervalo de confianza para el cociente de varianzas $\beta = \sigma_1^2/\sigma_2^2$ de dos distribuciones normales independientes con medias desconocidas:

$$\boxed{I_e = \left(\frac{S_1^2/S_2^2}{F_{n_1-1,n_2-1;\,\alpha/2}}, \frac{S_1^2/S_2^2}{F_{n_1-1,n_2-1;\,1-\alpha/2}}\right)} \quad \diamond\ \widehat{\beta} = \frac{S_1^2}{S_2^2} \quad \diamond\ \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim \frac{\dfrac{\chi^2_{n_1-1}}{n_1-1}}{\dfrac{\chi^2_{n_2-1}}{n_2-1}} \equiv F_{n_1-1,n_2-1}$$

- Intervalo de confianza para la diferencia media $\mu_D = \mu_X - \mu_Y$ de datos apareados (X_i, Y_i), $i = 1, ..., n$. Se supone que $D = X - Y \sim N(\mu_D, \sigma_D)$ con σ_D desconocida:

$$\boxed{I_e = \left(\overline{D} \pm t_{n-1;\,\alpha/2}\frac{S_D}{\sqrt{n}}\right)} \quad \diamond\ \widehat{\mu}_D = \overline{D} = \frac{1}{n}\sum_{i=1}^{n} D_i \quad \diamond\ \widehat{\sigma_D^2} = S_D^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(D_i - \overline{D}\right)^2$$

$$\diamond\ \frac{\overline{D} - \mu_D}{S_D/\sqrt{n}} \sim t_{n-1}; \quad D_i = X_i - Y_i\,,\ i = 1, ..., n; \quad D_1, ..., D_n\ m.a.s.\ \text{de}\ D = X - Y \sim N(\mu_D, \sigma_D)$$

- Intervalos de confianza para el parámetro p de una distribución binomial $X \sim B(n,p)$, n grande:

$$\boxed{I_a = \left(\widehat{p} \pm z_{\alpha/2}\sqrt{\frac{\widehat{p}\,(1-\widehat{p})}{n}}\right)} \qquad \boxed{I_a = \left(\frac{n\widehat{p} + \dfrac{z^2_{\alpha/2}}{2} \pm z_{\alpha/2}\sqrt{n\widehat{p}\,(1-\widehat{p}) + \dfrac{z^2_{\alpha/2}}{4}}}{n + z^2_{\alpha/2}}\right)}$$

$$\boxed{I_a = \left(sen^2\left(arcsen\sqrt{\frac{n\widehat{p} + 3/8}{n + 3/4}} \pm \frac{z_{\alpha/2}}{2\sqrt{n}}\right)\right)} \qquad \diamond\ \widehat{p} = \frac{X}{n} = \frac{N^{\underline{o}}\ total\ de\ éxitos}{N^{\underline{o}}\ total\ de\ pruebas}$$

$$\diamond\ X \sim B(n,p) \overset{a}{\approx} N\left(np, \sqrt{np(1-p)}\right) \qquad \diamond\ arcsen\sqrt{\frac{X + 3/8}{n + 3/4}} \overset{a}{\approx} N\left(arcsen\sqrt{p}, \frac{1}{2\sqrt{n}}\right)$$

$$\boxed{I_e = \left(\frac{X}{X + (n - X + 1)\,F_{2(n-X+1),2X;\,\alpha/2}}, \frac{(X+1)\,F_{2(X+1),2(n-X);\,\alpha/2}}{(n-X) + (X+1)\,F_{2(X+1),2(n-X);\,\alpha/2}}\right)}$$

- Intervalo de confianza para la diferencia de parámetros $w = p_1 - p_2$ de dos distribuciones binomiales independientes $X_1 \sim B(n_1, p_1)$ y $X_2 \sim B(n_2, p_2)$ con n_1 y n_2 grandes:

$$\boxed{I_a = \left((\widehat{p}_1 - \widehat{p}_2) \pm z_{\alpha/2} \sqrt{\frac{\widehat{p}_1 (1 - \widehat{p}_1)}{n_1} + \frac{\widehat{p}_2 (1 - \widehat{p}_2)}{n_2}} \right)} \qquad \diamondsuit \ \ \widehat{w} = \widehat{p}_1 - \widehat{p}_2 = \frac{X_1}{n_1} - \frac{X_2}{n_2}$$

$$\diamondsuit \ \ X_1 - X_2 \overset{a}{\approx} N\left(n_1 p_1 - n_2 p_2, \sqrt{n_1 p_1 (1 - p_1) + n_2 p_2 (1 - p_2)} \right)$$

- Intervalos de confianza para el parámetro λ de una distribución de Poisson $X \sim \mathcal{P}(\lambda)$, ($n\widehat{\lambda}$ grande):

$$\boxed{I_a = \left(\widehat{\lambda} \pm z_{\alpha/2} \sqrt{\widehat{\lambda}/n} \right)} \qquad \boxed{I_a = \left(\widehat{\lambda} + \frac{z^2_{\alpha/2}}{2n} \pm \frac{z_{\alpha/2}}{2} \sqrt{\frac{4\widehat{\lambda}}{n} + \frac{z^2_{\alpha/2}}{n^2}} \right)}$$

$$\boxed{I_a = \left(\left\{ \max\left(0, \sqrt{\widehat{\lambda} + \frac{3}{8n}} - \frac{z^2_{\alpha/2}}{2\sqrt{n}} \right) \right\}^2, \left\{ \sqrt{\widehat{\lambda} + \frac{3}{8n}} + \frac{z^2_{\alpha/2}}{2\sqrt{n}} \right\}^2 \right)}$$

$$\diamondsuit \ \ \sum_{i=1}^{n} X_i \sim \mathcal{P}(n\lambda) \overset{a}{\approx} N\left(n\lambda, \sqrt{n\lambda} \right) \qquad \diamondsuit \ \ \widehat{\lambda} = \overline{X} \overset{a}{\approx} N\left(\lambda, \sqrt{\lambda/n} \right)$$

$$\diamondsuit \ \ X \sim \mathcal{P}(\lambda) \overset{\lambda \text{ grande}}{\Longrightarrow} \sqrt{X + 3/8} \overset{a}{\approx} N\left(\sqrt{\lambda}, 1/2 \right) \qquad \diamondsuit \ \ \sqrt{n\widehat{\lambda} + 3/8} \overset{a}{\approx} N\left(\sqrt{n\lambda}, 1/2 \right)$$

$$\boxed{I_e = \left(\frac{\chi^2_{2K;1-\alpha/2}}{2n}, \frac{\chi^2_{2K+2;\alpha/2}}{2n} \right)} \qquad \diamondsuit \ \ K = \sum_{i=1}^{n} X_i$$

- Intervalos de confianza para la diferencia de parámetros $\delta = \lambda_1 - \lambda_2$ de dos distribuciones de Poisson independientes $X \sim \mathcal{P}(\lambda_1)$ e $Y \sim \mathcal{P}(\lambda_2)$, ($n_1\widehat{\lambda}_1$ y $n_2\widehat{\lambda}_2$ grandes):

$$\boxed{I_a = \left(\left(\widehat{\lambda}_1 - \widehat{\lambda}_2 \right) \pm z_{\alpha/2} \sqrt{\frac{\widehat{\lambda}_1}{n_1} + \frac{\widehat{\lambda}_2}{n_2}} \right)} \qquad \diamondsuit \ \ \widehat{\delta} = \widehat{\lambda}_1 - \widehat{\lambda}_2 \qquad \diamondsuit \ \ \begin{cases} \widehat{\lambda}_1 = \overline{X} \overset{a}{\approx} N\left(\lambda_1, \sqrt{\lambda_1/n_1} \right) \\ \widehat{\lambda}_2 = \overline{Y} \overset{a}{\approx} N\left(\lambda_2, \sqrt{\lambda_2/n_2} \right) \end{cases}$$

- Intervalo de confianza para el parámetro λ de una distribución exponencial $X \sim Exp(\lambda)$:

$$\boxed{I_e = \left(\frac{\chi^2_{2n;1-\alpha/2}}{2n\overline{X}}, \frac{\chi^2_{2n;\alpha/2}}{2n\overline{X}} \right)} \qquad \diamondsuit \ \ 2n\lambda\overline{X} = 2\lambda \sum_{i=1}^{n} X_i \sim G\left(\frac{2n}{2}, \frac{1}{2} \right) \equiv \chi^2_{2n}$$

$$\diamondsuit \ \ \widehat{\lambda} = \frac{n-1}{\sum_{i=1}^{n} X_i} \qquad \diamondsuit \ \ \text{Si } \theta = E[X] = 1/\lambda \Longrightarrow \widehat{\theta} = \overline{X}$$

- Intervalo de confianza para el cociente de parámetros $\omega = \lambda_1/\lambda_2$ de dos distribuciones exponenciales independientes $X \sim Exp(\lambda_1)$ e $Y \sim Exp(\lambda_2)$:

$$\boxed{I_e = \left(\left(\overline{Y}/\overline{X} \right) F_{2n_1,2n_2;1-\alpha/2}, \left(\overline{Y}/\overline{X} \right) F_{2n_1,2n_2;\alpha/2} \right)} \qquad \diamondsuit \ \ \widehat{\omega} = \frac{\overline{Y}}{\overline{X}}$$

$$\left. \begin{array}{l} 2n_1\lambda_1\overline{X} = 2\lambda_1 \sum_{i=1}^{n_1} X_i \sim G(n_1, 1/2) \equiv \chi^2_{2n_1} \\ 2n_2\lambda_2\overline{Y} = 2\lambda_2 \sum_{i=1}^{n_2} Y_i \sim G(n_2, 1/2) \equiv \chi^2_{2n_2} \end{array} \quad indep. \right\} \Longrightarrow \frac{\lambda_1/\lambda_2}{\overline{Y}/\overline{X}} \sim \frac{\frac{\chi^2_{2n_1}}{2n_1}}{\frac{\chi^2_{2n_2}}{2n_2}} \equiv F_{2n_1,2n_2}$$