MÉTODOS ESTADÍSTICOS DE LA INGENIERÍA

CONTRASTES DE HIPÓTESIS PARAMÉTRICAS

- Contrastes de hipótesis para la media μ de una distribución normal con varianza σ^2 conocida:

$$\begin{cases} H_0: & \mu = \mu_0 \\ H_1: & \mu \neq \mu_0 \end{cases} \quad R = \left\{ \mid Z \mid \geq z_{\alpha/2} \right\}$$

$$\diamondsuit \; Z = \frac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} \overset{H_0}{\sim} N(0,1)$$

$$\begin{cases} H_0: & \mu \leq \mu_0 \\ H_1: & \mu > \mu_0 \end{cases} \quad R = \{ Z \geq z_\alpha \}$$

$$\begin{cases} H_0: & \mu \geq \mu_0 \\ H_1: & \mu < \mu_0 \end{cases} \quad R = \{ Z \leq -z_\alpha \}$$

- Contrastes de hipótesis para la media μ de una distribución normal con varianza desconocida:

$$\begin{cases} H_0: & \mu = \mu_0 \\ H_1: & \mu \neq \mu_0 \end{cases} \quad R = \left\{ \mid T \mid \geq t_{n-1;\,\alpha/2} \right\}$$

$$\diamondsuit \; T = \frac{\overline{X} - \mu_0}{S / \sqrt{n}} \overset{H_0}{\sim} t_{n-1}$$

$$\begin{cases} H_0: & \mu \leq \mu_0 \\ H_1: & \mu > \mu_0 \end{cases} \quad R = \{ T \geq t_{n-1;\,\alpha} \}$$

$$\begin{cases} H_0: & \mu \geq \mu_0 \\ H_1: & \mu < \mu_0 \end{cases} \quad R = \{ T \leq -t_{n-1;\,\alpha} \}$$

- Contrastes de hipótesis para la varianza σ^2 de una distribución normal con media μ conocida:

$$\begin{cases} H_0: & \sigma = \sigma_0 \\ H_1: & \sigma \neq \sigma_0 \end{cases} \quad R = \left\{ W \notin \left(\chi^2_{n;\,1-\alpha/2} \;,\; \chi^2_{n;\,\alpha/2} \right) \right\}$$

$$\diamondsuit \; W = n \frac{D^2}{\sigma_0^2} \overset{H_0}{\sim} \chi^2_n$$

$$\begin{cases} H_0: & \sigma \leq \sigma_0 \\ H_1: & \sigma > \sigma_0 \end{cases} \quad R = \left\{ W \geq \chi^2_{n;\,\alpha} \right\}$$

$$\begin{cases} H_0: & \sigma \geq \sigma_0 \\ H_1: & \sigma < \sigma_0 \end{cases} \quad R = \left\{ W \leq \chi^2_{n;\,1-\alpha} \right\}$$

- Contrastes de hipótesis para la varianza σ^2 de una distribución normal con media desconocida:

$$\begin{cases} H_0: & \sigma = \sigma_0 \\ H_1: & \sigma \neq \sigma_0 \end{cases} \quad R = \left\{ W \notin \left(\chi^2_{n-1;\,1-\alpha/2} \;,\; \chi^2_{n-1;\,\alpha/2} \right) \right\}$$

$$\diamondsuit \; W = (n-1) \frac{S^2}{\sigma_0^2} \overset{H_0}{\sim} \chi^2_{n-1}$$

$$\begin{cases} H_0: & \sigma \leq \sigma_0 \\ H_1: & \sigma > \sigma_0 \end{cases} \quad R = \left\{ W \geq \chi^2_{n-1;\,\alpha} \right\}$$

$$\begin{cases} H_0: & \sigma \geq \sigma_0 \\ H_1: & \sigma < \sigma_0 \end{cases} \quad R = \left\{ W \leq \chi^2_{n-1;\,1-\alpha} \right\}$$

- Contrastes para la diferencia de medias $\gamma = \mu_1 - \mu_2$ de dos distribuciones normales independientes con varianzas σ_1^2 y σ_2^2 conocidas:

$$\begin{cases} H_0: & \mu_1 - \mu_2 = \gamma_0 \\ H_1: & \mu_1 - \mu_2 \neq \gamma_0 \end{cases} \quad R = \left\{ \mid Z \mid \geq z_{\alpha/2} \right\}$$

$$\diamondsuit \; Z = \frac{\left(\overline{X} - \overline{Y} \right) - \gamma_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \overset{H_0}{\sim} N(0,1)$$

$$\begin{cases} H_0: & \mu_1 - \mu_2 \leq \gamma_0 \\ H_1: & \mu_1 - \mu_2 > \gamma_0 \end{cases} \quad R = \{ Z \geq z_\alpha \}$$

$$\begin{cases} H_0: & \mu_1 - \mu_2 \geq \gamma_0 \\ H_1: & \mu_1 - \mu_2 < \gamma_0 \end{cases} \quad R = \{ Z \leq -z_\alpha \}$$

- Contrastes para la diferencia de medias $\gamma = \mu_1 - \mu_2$ de dos distribuciones normales independientes con varianzas desconocidas e iguales ($\sigma_1^2 = \sigma_2^2 = \sigma^2$):

$$\left\{ \begin{array}{ll} H_0: & \mu_1 - \mu_2 = \gamma_0 \\ H_1: & \mu_1 - \mu_2 \neq \gamma_0 \end{array} \right. \quad R = \left\{ \mid T \mid \geq t_{m;\,\alpha/2} \right\}$$

$$\diamondsuit\ T = \frac{\left(\overline{X} - \overline{Y}\right) - \gamma_0}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \overset{H_0}{\sim} t_m \ ; \ m = n_1 + n_2 - 2$$

$$\left\{ \begin{array}{ll} H_0: & \mu_1 - \mu_2 \leq \gamma_0 \\ H_1: & \mu_1 - \mu_2 > \gamma_0 \end{array} \right. \quad R = \{ T \geq t_{m;\,\alpha} \}$$

$$\left\{ \begin{array}{ll} H_0: & \mu_1 - \mu_2 \geq \gamma_0 \\ H_1: & \mu_1 - \mu_2 < \gamma_0 \end{array} \right. \quad R = \{ T \leq -t_{m;\,\alpha} \}$$

- Contrastes para la diferencia de medias $\gamma = \mu_1 - \mu_2$ de dos distribuciones normales independientes con varianzas desconocidas y distintas ($\sigma_1^2 \neq \sigma_2^2$):

$$\left\{ \begin{array}{ll} H_0: & \mu_1 - \mu_2 = \gamma_0 \\ H_1: & \mu_1 - \mu_2 \neq \gamma_0 \end{array} \right. \quad R \simeq \left\{ \mid T \mid \geq t_{f;\,\alpha/2} \right\}$$

$$\diamondsuit\ T = \frac{\left(\overline{X} - \overline{Y}\right) - \gamma_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \overset{H_0}{\approx} t_f$$

$$\left\{ \begin{array}{ll} H_0: & \mu_1 - \mu_2 \leq \gamma_0 \\ H_1: & \mu_1 - \mu_2 > \gamma_0 \end{array} \right. \quad R \simeq \{ T \geq t_{f;\,\alpha} \}$$

$$\left\{ \begin{array}{ll} H_0: & \mu_1 - \mu_2 \geq \gamma_0 \\ H_1: & \mu_1 - \mu_2 < \gamma_0 \end{array} \right. \quad R \simeq \{ T \leq -t_{f;\,\alpha} \}$$

- Contrastes para el cociente de varianzas $\beta = \sigma_1^2/\sigma_2^2$ de dos distribuciones normales independientes con medias μ_1 y μ_2 conocidas:

$$\left\{ \begin{array}{ll} H_0: & \beta = \beta_0 \\ H_1: & \beta \neq \beta_0 \end{array} \right. \quad R = \left\{ F \notin \left(F_{n_1,n_2;\,1-\alpha/2}, F_{n_1,n_2;\,\alpha/2} \right) \right\}$$

$$\diamondsuit\ F = \frac{D_1^2/D_2^2}{\beta_0} \overset{H_0}{\sim} F_{n_1,n_2}$$

$$\left\{ \begin{array}{ll} H_0: & \beta \leq \beta_0 \\ H_1: & \beta > \beta_0 \end{array} \right. \quad R = \{ F \geq F_{n_1,n_2;\,\alpha} \}$$

$$\left\{ \begin{array}{ll} H_0: & \beta \geq \beta_0 \\ H_1: & \beta < \beta_0 \end{array} \right. \quad R = \{ F \leq F_{n_1,n_2;\,1-\alpha} \}$$

- Contrastes para el cociente de varianzas $\beta = \sigma_1^2/\sigma_2^2$ de dos distribuciones normales independientes con medias desconocidas:

$$\left\{ \begin{array}{ll} H_0: & \beta = \beta_0 \\ H_1: & \beta \neq \beta_0 \end{array} \right. \quad R = \left\{ F \notin \left(F_{n_1-1,n_2-1;\,1-\alpha/2}, F_{n_1-1,n_2-1;\,\alpha/2} \right) \right\}$$

$$\diamondsuit\ F = \frac{S_1^2/S_2^2}{\beta_0} \overset{H_0}{\sim} F_{n_1-1,n_2-1}$$

$$\left\{ \begin{array}{ll} H_0: & \beta \leq \beta_0 \\ H_1: & \beta > \beta_0 \end{array} \right. \quad R = \{ F \geq F_{n_1-1,n_2-1;\,\alpha} \}$$

$$\left\{ \begin{array}{ll} H_0: & \beta \geq \beta_0 \\ H_1: & \beta < \beta_0 \end{array} \right. \quad R = \{ F \leq F_{n_1-1,n_2-1;\,1-\alpha} \}$$

- Contrastes para la diferencia de las medias $\mu_D = \mu_X - \mu_Y$ de dos distribuciones apareadas. Se supone que $D = X - Y \sim N(\mu_D, \sigma_D)$ con σ_D desconocida:

$$\left\{ \begin{array}{ll} H_0: & \mu_D = \mu_0 \\ H_1: & \mu_D \neq \mu_0 \end{array} \right. \quad R = \left\{ \mid T \mid \geq t_{n-1;\,\alpha/2} \right\}$$

$$\diamondsuit\ T = \frac{\overline{D} - \mu_0}{S_D/\sqrt{n}} \overset{H_0}{\sim} t_{n-1}$$

$$\left\{ \begin{array}{ll} H_0: & \mu_D \leq \mu_0 \\ H_1: & \mu_D > \mu_0 \end{array} \right. \quad R = \{ T \geq t_{n-1;\,\alpha} \}$$

$$\left\{ \begin{array}{ll} H_0: & \mu_D \geq \mu_0 \\ H_1: & \mu_D < \mu_0 \end{array} \right. \quad R = \{ T \leq -t_{n-1;\,\alpha} \}$$

- Contrastes para el parámetro p de una distribución binomial $X \sim B(n,p)$ con n grande:

$$\left\{ \begin{array}{ll} H_0: & p = p_0 \\ H_1: & p \neq p_0 \end{array} \right. \quad R \simeq \left\{ \mid Z \mid \geq z_{\alpha/2} \right\}$$

$$\diamondsuit\ Z = \frac{\widehat{p} - p_0}{\sqrt{p_0 (1 - p_0) / n}} \overset{H_0}{\approx} N(0,1)$$

$$\left\{ \begin{array}{ll} H_0: & p \leq p_0 \\ H_1: & p > p_0 \end{array} \right. \quad R \simeq \{ Z \geq z_\alpha \}$$

$$\left\{ \begin{array}{ll} H_0: & p \geq p_0 \\ H_1: & p < p_0 \end{array} \right. \quad R \simeq \{ Z \leq -z_\alpha \}$$

- Contrastes sobre la relación entre los parámetros p_1 y p_2 de dos distribuciones binomiales independientes $X_1 \sim B(n_1,p_1)$ y $X_2 \sim B(n_2,p_2)$; (n_1 y n_2 grandes; $\widehat{p} = (X_1 + X_2) / (n_1 + n_2)$):

$$\left\{ \begin{array}{ll} H_0: & p_1 = p_2 \\ H_1: & p_1 \neq p_2 \end{array} \right. \quad R \simeq \left\{ \mid Z \mid \geq z_{\alpha/2} \right\}$$

$$\diamondsuit\ Z = \frac{\widehat{p}_1 - \widehat{p}_2}{\sqrt{\widehat{p}(1-\widehat{p}) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} \overset{H_0}{\approx} N(0,1)$$

$$\left\{ \begin{array}{ll} H_0: & p_1 \leq p_2 \\ H_1: & p_1 > p_2 \end{array} \right. \quad R \simeq \{ Z \geq z_\alpha \}$$

$$\left\{ \begin{array}{ll} H_0: & p_1 \geq p_2 \\ H_1: & p_1 < p_2 \end{array} \right. \quad R \simeq \{ Z \leq -z_\alpha \}$$

- Contrastes para el parámetro λ de una distribución de Poisson $X \sim \mathcal{P}(\lambda)$, ($n\widehat{\lambda}$ grande):

$$\left\{ \begin{array}{ll} H_0: & \lambda = \lambda_0 \\ H_1: & \lambda \neq \lambda_0 \end{array} \right. \quad R \simeq \left\{ \mid Z \mid \geq z_{\alpha/2} \right\}$$

$$\diamondsuit\ Z = \frac{\widehat{\lambda} - \lambda_0}{\sqrt{\lambda_0 / n}} \overset{H_0}{\approx} N(0,1)$$

$$\left\{ \begin{array}{ll} H_0: & \lambda \leq \lambda_0 \\ H_1: & \lambda > \lambda_0 \end{array} \right. \quad R \simeq \{ Z \geq z_\alpha \}$$

$$\left\{ \begin{array}{ll} H_0: & \lambda \geq \lambda_0 \\ H_1: & \lambda < \lambda_0 \end{array} \right. \quad R \simeq \{ Z \leq -z_\alpha \}$$

- Contrastes sobre la relación entre los parámetros λ_1 y λ_2 de dos distribuciones de Poisson independientes $X \sim \mathcal{P}(\lambda_1)$ e $Y \sim \mathcal{P}(\lambda_2)$; ($n_1\widehat{\lambda}_1$ y $n_2\widehat{\lambda}_2$ grandes; $\widehat{\lambda} = \left(n_1\widehat{\lambda}_1 + n_2\widehat{\lambda}_2 \right) / (n_1 + n_2)$):

$$\left\{ \begin{array}{ll} H_0: & \lambda_1 = \lambda_2 \\ H_1: & \lambda_1 \neq \lambda_2 \end{array} \right. \quad R \simeq \left\{ \mid Z \mid \geq z_{\alpha/2} \right\}$$

$$\diamondsuit\ Z = \frac{\widehat{\lambda}_1 - \widehat{\lambda}_2}{\sqrt{\widehat{\lambda}/n_1 + \widehat{\lambda}/n_2}} \overset{H_0}{\approx} N(0,1)$$

$$\left\{ \begin{array}{ll} H_0: & \lambda_1 \leq \lambda_2 \\ H_1: & \lambda_1 > \lambda_2 \end{array} \right. \quad R \simeq \{ Z \geq z_\alpha \}$$

$$\left\{ \begin{array}{ll} H_0: & \lambda_1 \geq \lambda_2 \\ H_1: & \lambda_1 < \lambda_2 \end{array} \right. \quad R \simeq \{ Z \leq -z_\alpha \}$$

- Contrastes para el parámetro λ de una distribución exponencial $X \sim Exp(\lambda)$:

$$\left\{ \begin{array}{ll} H_0: & \lambda = \lambda_0 \\ H_1: & \lambda \neq \lambda_0 \end{array} \right. \quad R = \left\{ W \notin \left(\chi^2_{2n;1-\alpha/2}\ ,\ \chi^2_{2n;\alpha/2} \right) \right\}$$

$$\diamondsuit\ W = 2n\lambda_0\overline{X} = 2\lambda_0 \sum_{i=1}^{n} X_i \overset{H_0}{\sim} \chi^2_{2n}$$

$$\left\{ \begin{array}{ll} H_0: & \lambda \leq \lambda_0 \\ H_1: & \lambda > \lambda_0 \end{array} \right. \quad R = \left\{ W \leq \chi^2_{2n;1-\alpha} \right\}$$

$$\left\{ \begin{array}{ll} H_0: & \lambda \geq \lambda_0 \\ H_1: & \lambda < \lambda_0 \end{array} \right. \quad R = \left\{ W \geq \chi^2_{2n;\alpha} \right\}$$

- Contrastes para el cociente de parámetros $\omega = \lambda_1/\lambda_2$ de dos distribuciones exponenciales independientes $X \sim Exp(\lambda_1)$ e $Y \sim Exp(\lambda_2)$:

$$\left\{ \begin{array}{ll} H_0: & \lambda_1/\lambda_2 = \omega_0 \\ H_1: & \lambda_1/\lambda_2 \neq \omega_0 \end{array} \right. \quad R = \left\{ F \notin \left(F_{2n_1,2n_2;1-\alpha/2}, F_{2n_1,2n_2;\alpha/2} \right) \right\}$$

$$\diamondsuit\ F = \frac{\omega_0}{\overline{Y}/\overline{X}} \overset{H_0}{\sim} F_{2n_1,2n_2}$$

$$\left\{ \begin{array}{ll} H_0: & \lambda_1/\lambda_2 \leq \omega_0 \\ H_1: & \lambda_1/\lambda_2 > \omega_0 \end{array} \right. \quad R = \{ F \geq F_{2n_1,2n_2;\alpha} \}$$

$$\left\{ \begin{array}{ll} H_0: & \lambda_1/\lambda_2 \geq \omega_0 \\ H_1: & \lambda_1/\lambda_2 < \omega_0 \end{array} \right. \quad R = \{ F \leq F_{2n_1,2n_2;1-\alpha} \}$$